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THEORY AND MEASUREMENT OF OCULAR CHROMATIC ABERRATION

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Abstract—We have determined the transverse chromatic aberration of the human eye by measuring the apparent offset of a two-color vernier viewed foveally through a displaced, pinhole aperture. For the same subjects, we also determined the longitudinal chromatic aberration for foveal viewing by the method of best focus. In both cases, the results were closely predicted by a simple, reduced-eye optical-model for which transverse and longitudinal chromatic aberration are directly proportional, with the constant of proportionality being the amount of displacement of the pinhole from the visual axis. Further measurements revealed that the natural pupil was closely centered on the visual axis for two subjects and slightly displaced in the temporal direction for three other subjects. One implication of these results is that, although the eye has substantial chromatic aberration, the pupil is positioned so as to minimize the transverse component of the aberration for central vision, thereby optimizing foveal image quality for polychromatic objects.

Chromatic aberration   Schematic eye   Achromatic axis

INTRODUCTION

Over 300 years ago, Newton described a simple experiment to demonstrate the presence of chromatic aberration in the human eye (Newton, 1670). An opaque card is held near the eye so that only the rays of light passing close to one edge of the pupil are allowed to enter the eye. This maneuver isolates incident light rays which strike the refracting surfaces of the eye obliquely and thus are strongly refracted. Because the indices of refraction of the ocular media vary inversely with wavelength, blue rays will be refracted more than red rays. One consequence of this chromatic dispersion, as Newton described, is that the edge of a white object will not be seen distinctly, but will be tinged with color.

Modern accounts of ocular chromatic aberration distinguish between two manifestations of chromatic dispersion according to their distinctly different effects on the retinal image. Longitudinal chromatic aberration is the variation of the eye's focusing power for different wavelengths. For the adult human eye (nominal focussing power = 60 D), this chromatic difference of focus amounts to approx. 2 D over the visible spectrum from 400 to 700 nm (Wald & Griffin, 1947). Transverse chromatic aberration, on the other hand, is a variation of image location for different wavelengths. This chromatic difference of position causes the image of an off-axis, white point source to be spread out across the retina as colored fringes. By the same mechanism, the retinal image of an extended object centered on the optical axis of the eye will be slightly smaller for the blue rays than for the red rays. Consequently, transverse chromatic aberration is often described as a variation of magnification with wavelength which has been calculated to be less than 1% (Bennett & Rabbetts, 1984). Although such a small value may seem insignificant at first glance, we will provide both theoretical and experimental evidence of its large visual effect.

The importance of chromatic aberration for retinal image quality stems from its dual effect on the modulation transfer function of the eye. For a localized patch of monochromatic grating, the longitudinal aberration affects image contrast through the mechanism of defocus whereas the transverse aberration affects image phase through the mechanism of displacement. Consequently, the various wavelength components of a polychromatic grating will be imaged with different amounts of contrast attenuation and phase shift. When superimposed on the retina, the resulting image will be a grating with net contrast and phase which depends in a complex way on stimulus parameters and the optical properties of the eye. In order to extend these qualitative arguments into quantitative predictions, a realistic optical model of ocular chromatic abberation is required. Thus, one motivation for the present study was to develop a model which could form the basis for further, detailed analysis of the combined effects of longitudinal and transverse chromatic aberrations on retinal image quality.

Longitudinal chromatic aberration is now well understood as a result of numerous experimental studies (Ivanoff, 1946, 1947, 1953; Wald & Griffin, 1947; Bedford & Wyszecki, 1957; Jenkins, 1963; Charman & Jennings, 1976; Powell, 1981; Lewis, Katz & Oehrlein, 1982; Ware, 1982; Mandelman & Sivak, 1983; Sivak & Millodot, 1974; Howarth & Bradley, 1986). The consensus view is that the eye's longitudinal abberation conforms to theoretical expectations based on the dispersion of the ocular media. However, comparatively little is known about the magnitude of transverse chromatic abberation of human eyes. This may be because the fovea is relatively close to the optical axis of the eye (Emsley, 1952) and consequently the aberration for central vision is small and difficult to measure (Hartridge, 1947). Theoretically, the magnitude of the aberration should increase as the visual stimulus is moved further into the peripheral field (because of the increased angle of incidence of light rays) but at the same time our ability to appreciate the dispersion of the spectrum diminishes because of the relatively poor spatial acuity (Wertheim, 1894) and color sensitivity (Van Esch, Koldenhoff, Van Doorn & Koenderink, 1984) of peripheral vision. Consequently, it has proved difficult to systematically investigate the variation of transverse chromatic aberration with stimulus eccentricity. We are aware of only one published attempt to do so (Ogboso & Bedell, 1987), and this gave equivocal results.

Recently, we have calculated the expected magnitude of transverse chromatic aberration and have found that it is potentially a major factor limiting image quality and visual performance in circumstances where polychromatic light rays strike the refracting surfaces of the eye obliquely (Howarth, 1984; Thibos, 1987). Preliminary evidence in support of these theoretical predictions has been obtained from measurements of visual acuity for white gratings formed on the retina interferometrically, a technique which avoids the contrast-reducing effects of defocus and thus isolates the effects of the transverse aberration. These results indicate that transverse chromatic aberration alone can reduce visual acuity threefold when rays are confined to enter the eye near the margin of the pupil (Thibos, Bradley, Still & Henderson, 1987a). However, Ogboso and Bedell (1987) concluded from their experimental measurements that the transverse chromatic aberration of the human eye does not agree with theoretical expectations.

Given the inconsistencies of published accounts, the paucity of experimental data, and the need for a sound theoretical model, we have renewed the effort to measure the magnitude of the eye's transverse aberration. To avoid the experimental limitations imposed by peripheral vision we have employed Newton's original method of observing the effects of the aberration with foveal vision through a displaced, pinhole aperture. For this eye + pinhole optical system, the transverse aberration was measured psychophysically with a two-color, vernier-alignment technique introduced some years ago by Le Grand and Ivanoff (Ivanoff, 1946).

A survey of the literature and textbooks in the field of visual optics reveals a lack of consensus for the definitions of terms which apply to ocular chromatic aberration and the absence of a clear conceptual model to guide experimental design and analysis of results. We therefore precede the experimental portion of this report with the development of a simple optical model of the eye which predicts quantitatively both the transverse and longitudinal components of ocular chromatic aberration. A key element of the theoretical treatment is the identification of the path of the chief nodal ray as an achromatic reference axis of central importance. Using the conceptual framework embodied in the model, we proceed to examine several related questions. First, what is the relationship between transverse and longitudinal chromatic aberration of the eye? Second, what is the relationship between the magnitude of transverse aberration induced by a displaced pinhole aperture and the amount of transverse aberration ordinarily present for peripheral vision through the natural pupil? Third, how may one determine experimentally the relationship between the achromatic and visual axes of the eye? Fourth, is the pupil displaced from the visual axis in the normal eye and, if so, how much transverse chromatic aberration does that displacement induce for foveal vision?

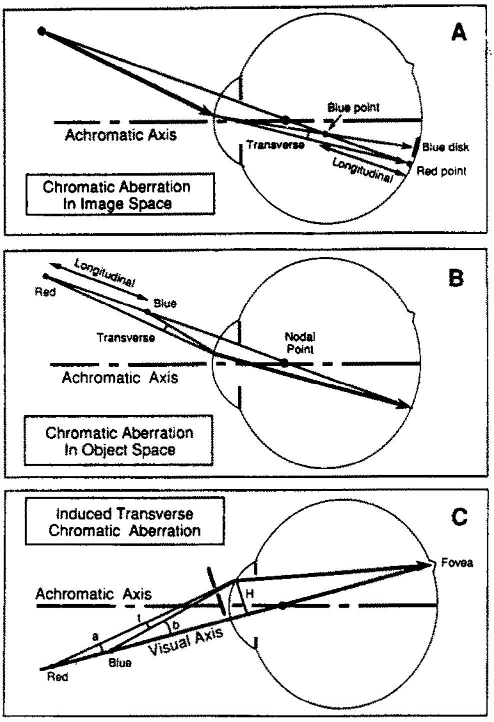

Fig. 1. Schematic illustration of transverse and longitudinal chromatic aberration of the eye specified in image space (A), object space (B) and for foveal vision through an artificial pupil (C). Chief rays are shown only for the short- and long-wavelength ends of the visible spectrum. Drawings are not to scale.

THEORY AND DEFINITIONS

Ocular chromatic aberration

Chromatic aberration of the eye may be defined with reference to the ray diagram of Fig. 1A. The eye is represented schematically by a modification of Emsley's (1952) version of the traditional, reduced-eye model, the parameters of which were fixed to conform with Gullstrand's schematic eyes. Optically, the reduced eye is a volume of water with a single, spherical, refracting surface. The nodal point of this model lies at the geometrical center of the refracting surface and this is invariant with wavelength. Emsley's model is appealing for its simplicity but lacks a pupil and has no intrinsic optical axis. Recognizing the central importance of the pupil for an analysis of transverse chromatic aberration of the eye, Thibos (1987) suggested the model be modified to include a limiting aperture positioned 3.63 mm in front of the nodal point, which corresponds to the location of the iris and anterior surface of the lens in Gullstrand's simplified schematic eye. By including the pupil, a new reference axis emerges as the line containing the center of the pupil and the nodal point. For reasons which will become apparent further on, we will refer to this reference line as the *achromatic axis* of the reduced eye model.

Because the refractive index of water decreases as wavelength increases, light rays entering the water medium of the reduced eye from air will be refracted less for long (red) wavelengths than for short (blue) wavelengths. This *chromatic dispersion* of the incident beam has two important consequences for the optical performance of the model eye. First, the image of the source formed by blue rays will be in front of the image formed by red rays. This chromatic difference of focus is called *longitudinal chromatic aberration* and may be specified by the distance between image planes for different wavelengths. Second, the chief rays for red and blue light emitted by a source located off the achromatic axis will strike the retinal surface at different places, where the *chief ray* of a point source is that ray which passes through the center of the pupil. The chromatic difference of position is called *transverse* (or lateral) *chromatic aberration* and may be specified by the angle between the refracted chief rays for different wavelengths.

Unlike longitudinal chromatic aberration, the transverse chromatic aberration of the eye depends critically on object location in the visual field and pupil position within the eye. Pupil location is important because it selects that pencil of the incident ray bundle which passes on to stimulate the retina. Object location is important because it determines the angle of incidence of these selected rays. By Snell's law of refraction, the angle of incidence determines the amount of chromatic dispersion and thus the location of the retinal images for different wavelengths. Determining image location for a given wavelength is problematical because the defocussing effect of longitudinal chromatic aberration produces a blurred retinal image of each point of the object. We deal with this problem in the usual way by assigning the center of the blur circle as the image position (Westheimer, 1972). According to geometrical optics, the center of the blur circle for an optical system suffering only from a defect of focus is found by tracing that ray which passes through the center of the pupil, i.e. the chief ray.

For a given pupil location, each object point in the visual field gives rise to a chief ray which passes through the pupil center. Of the infinite number of chief rays thus conceivable, only one passes through the nodal point and thus avoids the dispersive effects of refraction. Consequently, the path of this *chief nodal ray* has special significance in the present context because it locates the image of that unique object point which is subject to zero transverse chromatic aberration. This is the fundamental reason for identifying the path of the chief nodal ray (i.e. the line connecting the center of the pupil and the nodal point) as the achromatic axis of the eye.* We note that, in general, the achromatic axis would not be expected to intersect the retina at the fovea. However, the achromatic axis could be redirected to the fovea, or other peripheral loci, by the introduction of an external, limiting aperture. Thus, from an experimental point of view, the achromatic axis is an important reference because it is operationally defined and subject to manipulation while from a theoretical point of view the achromatic axis is important because it identifies that axis which nullifies transverse chromatic aberration. Hence, the achromatic axis establishes a vital link between optical theory and real eyes.

Instead of representing ocular chromatic aberration in image space as described above, an experimental study may be served best by representing the aberration in object space as shown in Fig. 1B. That is, instead of having a single polychromatic object and multiple images (one for each wavelength of light present) there will be multiple objects (each of a different wavelength) forming a single polychromatic image. Using a similar line of reasoning as above, for a red and blue object to be focussed on the retina simultaneously, the blue object must be placed closer to the eye. Now the longitudinal aberration is quantified by the distance between objects or, more commonly, by their dioptric interval (i.e. difference of inverse distances from the refracting surface). Since dispersion will cause the blue chief ray to be refracted more than the red chief ray, the blue ray must strike the refracting surface at a greater angle in order to form a single image. In this case, transverse chromatic aberration may be specified by the angle between the incident chief rays of the red and blue objects (Howarth, 1984).

Induced transverse chromatic aberration

Experimental manipulation of the amount of transverse chromatic aberration affecting the retinal image can be achieved by varying the angle of incidence of the chief ray. One method would be to vary the object's location within the visual field, but this presents a practical problem because the poor spatial acuity of peripheral vision leads to inaccurate judgments of the spatial coincidence of a pair of targets, especially if one or both is out of focus because of longitudinal chromatic aberration. Another way is to leave the stimulus in one place (e.g. at the fixation point) and vary the achromatic axis by interposing a pinhole aperture. This method of foveal viewing through a displaced pinhole, employed originally in Newton's demonstration of chromatic aberration of the human eye and later developed by Ivanoff (1946), is illustrated schematically in Fig. 1C. We chose this method for the present study because it avoids the limitations associated with peripheral vision. To avoid possible confusion of terms, the word pupil will be reserved for the eye's natural aperture and the word pinhole for the artificial aperture placed in front of the eye. Further on we will show that the two arrangements illustrated in Fig. 1B and C are optically equivalent for the reduced-eye model, which suggests that present results may prove useful for estimating the magnitude of transverse chromatic aberration in normal peripheral vision.

The magnitude of transverse chromatic aberration induced by manipulating the pinhole location is specified by the angle between the incident chief rays for the red and blue fixation targets. Traditionally, the chief ray from a fixation target is called the *line-of-sight*, and so by this terminology the transverse chromatic aberration present for foveal viewing through a pinhole aperture is given by the angle between the red and blue lines-of-sight. Another important reference line shown in Fig. 1C is the *visual axis*, which is traditionally defined as the line joining the fixation target, nodal point and

*Ivanoff (1953) coined the term achromatic axis to represent the path of an undeviated ray from a fixation target to the fovea, not necessarily passing through the center of the pupil. We show further on that Ivanoff's concept might be more usefully interpreted as an operational definition of the visual axis. By redefining the achromatic axis as proposed here, it assumes the role of a primary reference axis in the theory of ocular chromatic aberration. For more elaborate schematic eyes, the achromatic axis will have two segments. In object space, this axis joins the center of the entrance pupil to the anterior nodal point and in image space it joins the center of the exit pupil to the posterior nodal point. In this case we presume the variation of nodal points with wavelengths is insignificant.

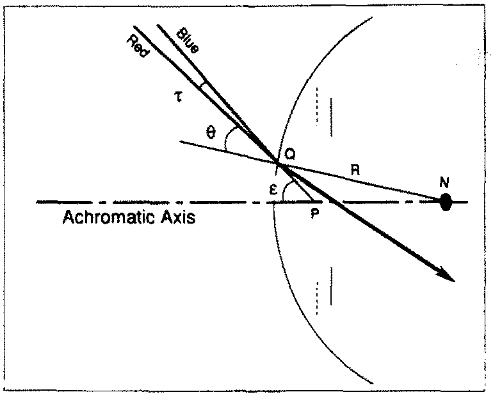

Fig. 2. Geometrical optics of transverse chromatic aberration in object space for the reduced eye. By Snell's law, the magnitude of the aberration (τ) varies directly with the angles of incidence (θ) of the chief rays, which in turn depend on stimulus eccentricity (ε) and the distance from the nodal point (N) to the center of the entrance pupil (P) (detail of Fig. 1B).

fovea. Because of the simplicity of our optical model of the eye, it is possible to derive an exact solution for the magnitude of the transverse aberration as a function of pinhole displacement from the visual axis. Our solution, which was used to predict the experimental results, is provided in the Appendix.

Only one of the three points defining the visual axis is located outside the eye. From a practical viewpoint, it is useful to establish a second external point, thus locating the visual axis in object space. To do this we note that if the pinhole in Fig. 1C were to be centered on the visual axis, then the lines-of-sight, the achromatic axis of the eye + pinhole optical system, and the visual axis would all coincide. Consequently, we may operationally define the visual axis as the line containing object point and pinhole center when the pinhole is positioned so as to nullify the eye's transverse chromatic aberration for foveal viewing. Further on (see Methods) we describe a simple psychophysical technique for subjective determination of this achromatic condition.

An equivalence relationship for induced and natural transverse chromatic aberration

For the reduced-eye model, the two ray diagrams of Fig. 1B and C, shown in greater detail in Figs 2 and 3, would be congruent if the angles of incidence for the red and blue chief rays were the same in the two diagrams. That is, if $\phi = \theta$ then $t = \tau$. Thus one may think of the pinhole as a device which induces transverse chromatic aberration for foveal vision through a decentered, artificial aperture which is equal to that ordinarily present when a peripheral object of some equivalent eccentricity is seen through a centered, natural pupil. An explicit statement of this equivalence is developed next.

According to the law of sines as applied to triangle NPQ in Fig. 2:

$$\sin(\theta) = (NP/R) \times \sin(\varepsilon); \qquad (1)$$

where $NP = 3.98$ mm (Thibos, 1987) is the distance from nodal point to center of the entrance pupil and $R = 5.55$ mm (Emsley, 1952) is the radius of curvature of the refracting surface. In the alternative case of foveal vision through a pinhole displaced by distance H from the visual axis (Fig. 3), the lines-of-sight are nearly parallel to the visual axis for distant fixation targets and so, to close approximation:

$$\sin(\phi) = H/R. \qquad (2)$$

Equating angles θ and ϕ we obtain from equations (1) and (2) the desired equivalence relationship between stimulus eccentricity (ε) and pinhole displacement (H):

$$H = NP \times \sin(\varepsilon). \qquad (3)$$

By this approximate equation, each millimeter of displacement of the pinhole induces the same amount of transverse chromatic aberration as does 15 deg of stimulus eccentricity.

The relationship between longitudinal and transverse chromatic aberration

Given the above definitions and the proposed model of the eye's optical system, it is possible

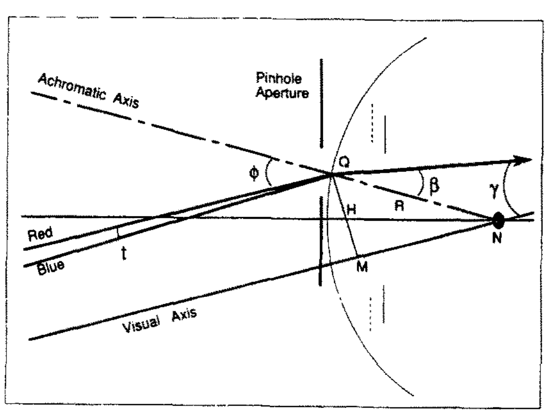

Fig. 3. Geometrical optics of induced transverse chromatic aberration for foveal viewing through a displaced pinhole. Displacement (H) of the pinhole from the visual axis changes the direction of the achromatic axis of the eye + pinhole optical system, increases the angles of incidence of the chief rays (ϕ), and thus induces the transverse aberration (t) (detail of Fig. 1C).

to reveal the simple relationship which exists between longitudinal and transverse chromatic aberrations for foveal viewing through a displaced pinhole. Using the following geometrical proof, which closely follows that of Ivanoff (1953), it is shown that, to first approximation, these two facets of chromatic aberrations are directly proportional, with the constant of proportionality being the displacement of the pinhole from the visual axis. With reference to Fig. 1C, let the distances from the eye to the red and blue conjugate points be A and B, respectively. Then

$$\text{Longitudinal aberration} = L = 1/B - 1/A. \quad (4)$$

Let the angles made by the red and blue incident chief rays with the visual axis be a and b, respectively. Then the angle t between incident chief rays is:

$$\text{Transverse aberration} = t = b - a \quad (5a)$$

$$= H/B - H/A. \quad (5b)$$

The transition from (5a) to (5b) employs the small angle approximation $x = \tan(x)$ which introduces negligible error ($< 1\%$) for pinhole displacements up to 4 mm from the visual axis. Combining equations (4) and (5b) gives the desired proportional relationship:

$$t = H \times L. \quad (6)$$

If distances H, A, and B are specified in meters then angle t is in radians and L is in diopters. It should be noted that the model's longitudinal aberration L is not constant but varies slightly with H because of the spherical aberration of the refracting surface. As the pinhole becomes more displaced. the conjugate points move closer to the refracting surface and their dioptric interval increases. Thus the ratio t/H increases with pinhole displacement, as illustrated by the nonlinear theoretical curves in Fig. 4. For the wavelengths of our experiments (433, 622 nm), L increases only 9% over the first 2 mm of pinhole displacement, from a paraxial value of 1.14 to 1.24D, but increases 67% to 1.90D at 4 mm of displacement from the visual axis.

METHODS

Transverse chromatic aberration

In principle. our experimental method is the same as Ivanoff's method of chromatic parallax (Ivanoff, 1946, 1953; Le Grand, 1967; Bennett & Rabbetts, 1984). However, our apparatus is somewhat simpler and has the advantage of containing no optical components between stimulus and observer which might spuriously introduce chromatic aberration and thus confound the measurement of the eye's chromatic aberration with equipment aberration. In summary, a pinhole aperture was used to manipulate the angle of incidence of the foveal chief ray and so vary the amount of transverse chromatic aberration affecting a fixation stimulus. The magnitude of the aberration was measured as a function of pinhole location using a two-color, vernier-alignment task.

Subjects used their right eye to view the stimulus through a pinhole of 0.5 mm diameter. The target field was located 3 m from the subject and consisted of a 2×2 deg sheet of diffusing glass transilluminated by light from a slide projector. A calibrated sandwich of interference and gelatin filters was imaged by the projector onto the diffusing glass in such a way that the upper half of the field was red and the lower half was blue. Mean luminance was 106 cd/m^2 (red) and 16 cd/m^2 (blue), the spectral bandwidth at half-height was 10 nm (red) and 18 nm (blue) and the mean wavelength (i.e. central moment of the luminance spectrum) was 622 nm (red) and 433 nm (blue). Immediately in front of the glass were two vertical, black rods (1 deg $\times$ 7 min) one above the other which served as a vernier alignment target. The top rod was fixed and the bottom rod could be displaced horizontally by the subject using a machinist's micrometer (resolution = 2 sec arc). To place the micrometer within the subject's reach, the optical path was folded with a large, front-surface mirror. The border between the upper and lower halves of the field was masked by another black rod of the same width. Each of the authors served as subjects and the sixth subject (MC) was recruited from the study by Simonet and Campbell (1990). The presence of significant myopia for three of the subjects (DS = 4.5D, PH = 4D, XZ = 2D) and chromatic errors of focus for everyone was not a major handicap because of the great depth of field provided by the pinhole aperture.

The subject's head was held firmly in place by the combination of a bite bar and forehead rest. This rigid assembly was mounted in a machinist's vise, allowing the experimeter to translate the subject's head orthogonally to the line-of-sight, both vertically and horizontally to the nearest 0.05 mm, behind the fixed pinhole. All data reported here were collected for a horizontal traverse of the pupil. First we set the vertical position of the head so that the pinhole was midway between the upper and lower margins of the pupil and then translated the head horizontally until the pinhole was near the left (nasal) side of the pupil. All subsequent positions of the head were referenced to this initial starting position. The subject's task was to adjust the location of the lower rod until it appeared to be aligned with the upper rod. The actual location of the lower rod was recorded for 5 replications of the task and the mean offset was expressed as a visual angle. The experimenter then shifted the subject's head horizontally by 1.28 mm (0.05 in) and the process was repeated until the subject's view of the target through the pinhole became obscured by the iris. Then the head was shifted 0.64 mm in the opposite direction and the entire experiment repeated in the reverse direction. In this way, two traverses of the pinhole across the pupil were interleaved with measurements spaced at 0.64 mm intervals. Although we chose to keep the pinhole fixed and moved the head, the results will be described in terms of pinhole relative to the eye.

In order to obtain the largest possible range of displacement for the pinhole, we dilated the subject's pupil and paralyzed accommodation (0.5% cyclopentalate hydrochloride). Unfortunately, this pharmacological agent is known to induce asymmetrical, non-physiological dilation in some individuals (Walsh, 1988), which would preclude an assessment of pinhole location relative to the center of the natural pupil. To avoid this potential problem, the location of the pinhole was referenced to the visual axis, as operationally defined in Theory, which is also the achromatic axis of the eye + pinhole optical system for foveal viewing. By this convention, the zero value of the abscissae of Figs 4 and 5 corresponds to that pinhole position for which the vernier target, when physically aligned, also appeared aligned to the subject. Since our experimental design was based on a predetermined sequence of head positions, it was unlikely that one of the set positions would by chance put the pinhole exactly on the visual axis and thus perfectly nullify transverse chromatic aberration. Therefore, we calculated the pinhole position that would have yielded zero vernier offset by linear interpolation of the 5 or 6 mean values for which vernier offset was nearly zero. We then subtracted that interpolated position from all other measurements of pinhole position.

Longitudinal chromatic aberration

A conventional Badal optometer described in detail elsewhere (Howarth & Bradley, 1986) was used to determine the plane of best focus for a pair of thin, vertical human hairs back-illuminated by light passed through the same set of filters used in the transverse aberration experiment described above. Using method of adjustment, a minimum of 5 settings were obtained for each wavelength of light. As in the transverse aberration experiment, the pupil was dilated and accommodation paralyzed (0.5% cyclopentalate hydrochloride).

RESULTS

The magnitude of transverse chromatic aberration

Our main experimental results quantify the following observation. When the black bars of a two-color vernier target are physically aligned, they appear to the observer to be misaligned. This apparent misalignment is greatest when the pinhole aperture is placed near the margin of the eye's pupil. The direction of the vernier offset obeys the following rule: the bar in the blue field moves with the pinhole. For example, when the pinhole is displaced towards the temporal side of the eye, the bar in the blue field appears to be more temporal than the bar in the red field. To compensate, the subject moves the bar in the blue field nasally until the vernier target appears aligned. The resulting physical misalignment of the target for this example is illustrated by Fig. 3 and was recorded during the experiment as a visual angle t, which we interpreted according to the definitions in Theory as the transverse chromatic aberration of the eye + pinhole optical system. Sign conventions are arbitrary and in this paper both the visual angle and pinhole displacement are taken as positive for the preceding example. Since pinhole location is specified relative to the visual axis (see Methods), zero displacement, by definition, induces zero transverse chromatic aberration.

The transverse aberration induced by a displaced pinhole increased almost linearly with displacement up to a maximum value of about 1/3 deg, as shown for our six subjects in Fig. 4. Vernier settings were highly repeatable as may be judged from the relatively small error bars, which represent $\pm$ 1 SD of 5 settings. The average standard deviation for the 12 or so pinhole positions tested was less than 1 min for each

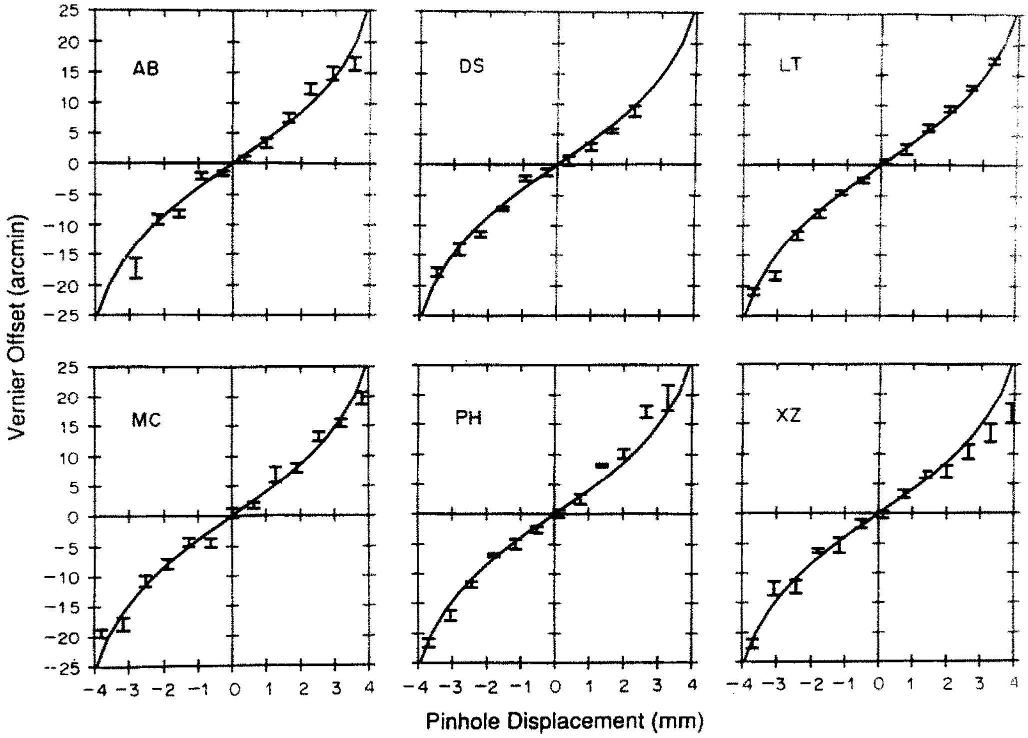

Fig. 4. Experimental determination of the magnitude of transverse chromatic aberration for six subjects. Ordinate is the offset of a two-color vernier target required by the subject to achieve subjective alignment, as a function of pinhole displacement from the visual axis shown on abscissa. Sign convention is described at beginning of Results. Error bars shown ± 1 SD ($n = 5$) from the mean results. Solid curves are the theoretical predictions of the reduced eye as computed by the algorithm defined in the Appendix.

subject, as reported in column 8 of Table 1. Individual standard deviations at any pinhole location rarely exceeded 1 min, and were typically 0.5 min for more central positions of the pinhole. We conclude from these results that, for most locations of the pinhole, the magnitude of the induced transverse aberration greatly exceeded threshold vernier acuity. In the most extreme cases, vernier offsets were more than an order of magnitude greater than vernier acuity for our subjects under the conditions of these experiments.

Also shown in Fig. 4 are the theoretical predictions of transverse chromatic aberration computed by the algorithm described in the Appendix for the model eye proposed in Theory. Visual inspection of the figures reveals a close match between theory and measurement, except at the very margins of the pupil for some subjects. We checked for consistent deviations from the model by pooling the data from our six subjects as shown in Fig. 5. The symbols appear equally scattered about the theoretical curve without any clear tendency to lie either above or below the prediction. We conclude from this result that the reduced-eye model is not grossly deficient and, to first approximation, would adequately describe the transverse chromatic aberration induced by a pinhole aperture over the full excursion of an 8 mm diameter pupil.

The foregoing conclusions were confirmed quantitatively by the following statistical assess-

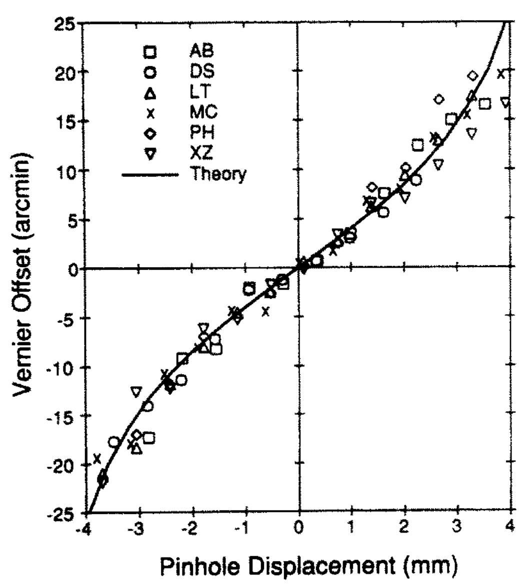

Fig. 5. Comparison of experimental results for the six subjects of Fig. 4. Symbols show mean ($n = 5$) results; error bars are omitted for clarity. Solid curve is the theoretical prediction for the reduced-eye model.

Table 1. Summary of mean experimental results (± 1SEM) for six subjects. From left to right, columns show (1) subject's initials, (2) slope of paraxial portion of data curves in Fig. 4, (3) slope in units of diopters, (4) measurement of longitudinal chromatic aberration, (5) measured offset of natural pupil from visual axis, (6) calculated value of angle ψ between visual and achromatic axes, (7) calculated value of transverse chromatic aberration for foveal viewing through natural pupil, (8) vernier acuity as estimated by the average half-length of error bars in Fig. 4.

1 Subject	2 Slope (min/mm)	3 Slope (diopters)	4 Best focus (diopters)	5 Pupil shift (mm)	6 Angle ψ (deg)	7 Foveal TCA (min)	8 Vernier (min)
AB	4.40 ± 0.22	1.28 ± 0.06	1.18 ± 0.02	0.14 ± 0.04	2.02 ± 0.58	0.62 ± 0.18	0.85
DS	3.69 ± 0.16	1.07 ± 0.05	1.77 ± 0.05	0.22 ± 0.05	3.17 ± 0.72	0.82 ± 0.19	0.62
LT	4.19 ± 0.14	1.22 ± 0.04	1.55 ± 0.01	0.06 ± 0.05	0.86 ± 0.72	0.31 ± 0.21	0.53
MC	4.50 ± 0.34	1.31 ± 0.10	1.43 ± 0.07	0.37 ± 0.08	5.33 ± 1.15	1.67 ± 0.36	0.82
PH	4.98 ± 0.24	1.45 ± 0.07	1.74 ± 0.08	n/a	n/a	n/a	0.80
XZ	4.50 ± 0.20	1.31 ± 0.06	1.70 ± 0.03	−0.08 ± 0.04	−1.15 ± 0.58	−0.36 ± 0.18	0.97
Mean	4.38 ± 0.17	1.27 ± 0.05	1.56 ± 0.09	0.14 ± 0.08	2.05 ± 1.09	0.61 ± 0.33	0.77
Theory	3.88	1.14	1.14				

ment of goodness-of-fit of the model to the data. Analysis of variance was performed separately for each subject using the mean data. The results indicated that the model accounts for most of the correlation of vernier offset with pinhole displacement and that the small residual deviation of the data from the model is insignificant ($P < 0.001$ for every subject). Thus we have no basis for rejecting the model based on this initial analysis of mean results. A more searching test of the model was then performed by analyzing the variance of the raw data from the model. In this case, the variance of the replicated ($n = 5$) settings from the individual means of each data set provides an independent estimate of measurement variability. This analysis revealed that the deviation of the raw data from the model was significantly greater than could be accounted for by the extremely low variance of the individual data sets. In other words, the small discrepancies between mean results and theoretical curves in Fig. 4 cannot be attributed to poor performance at vernier alignment. Thus some other source of variability must be responsible for the residual error in the model's predictions. Two possibilities are suggested in Discussion.

The slopes of the central, linear portion of the data curves were estimated by least-squares regression of the data obtained for the most central 5 or 6 locations of the pinhole (i.e. pinhole displacement less than 1.6 mm from the visual axis) and the results are given in column 2 of Table 1. Summarizing the mean results for our six subjects, we found that the transverse chromatic aberration for our selected wavelengths (433 nm, 622 nm) increased at the average rate of 4.4 min per mm of pinhole displacement. Expressing this result in units of radians per meter (i.e. diopters, column 3 of Table 1) corresponds to an average slope of 1.3 D, which is slightly greater than the theoretical expectation of 1.1 D for the reduced eye model. This small difference is consistent with the comparison between real eyes and the water model presented by Wald and Griffin (1947).

Transverse chromatic aberration for foveal vision through the natural pupil

Shortly after the above experiments were completed, we become aware of the work of Simonet and Campbell (1988, 1990) who have independently been studying transverse chromatic aberration for foveal vision. They have emphasized the significance of pupil displacement from the visual axis for determining the magnitude of the aberration under natural viewing conditions, a factor not directly accessible to us because of our drug induced, possibly asym-

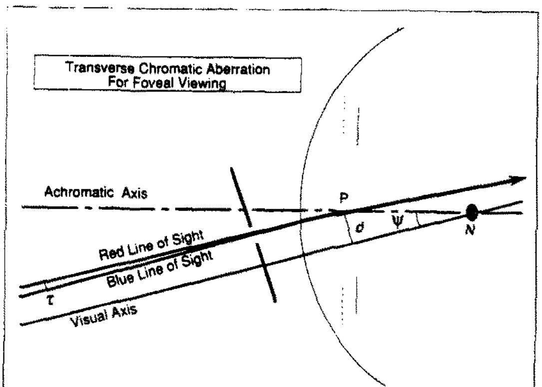

Fig. 6. Foveal transverse chromatic aberration caused by displacement of the natural pupil from the visual axis. The angle ψ between the visual and achromatic axes may be determined experimentally from d, the displacement of a pinhole aperture required to isolate the natural line of sight.

metrical dilation of the iris (see Methods). We are indebted to them for pointing out, however, that our measurements of how transverse chromatic aberration varies with pinhole displacement can be used to deduce the expected magnitude of the aberration for natural viewing through the normal pupil. The experimental rationale is illustrated in Fig. 6, which shows the pinhole displaced d mm from the visual axis in order to isolate those rays which pass through the center of the undilated pupil. This arrangement thus establishes the lines-of-sight not only for foveal vision through the pinhole, but also for normal foveal viewing when the pinhole is removed. Therefore, the magnitude of the aberration measured in the former case should apply equally to the latter.

Since we know from Fig. 4 how transverse chromatic aberration varies with pinhole displacement, it only remained to measure d for the undilated iris of our subjects. We did this in a separate experiment in which cycloplegic drugs were not used. First, the two-color vernier target was set to be physically aligned and the subject was instructed to position the pinhole so that the vernier target appeared aligned. According to our operational definition, this located the position of the visual axis. Next, the subject translated the pinhole horizontally to find the leftmost and rightmost positions for which the target was just visible without being obscured by the iris. The difference of these two positions was taken as an estimate of pupil diameter whereas their mean provided an estimate of the geometrical center of the undilated pupil and thus established one measurement of d. This entire measurement sequence was repeated 10 times, alternating left and right starting positions, and the resulting mean values for d are given in column 5 of Table 1. Throughout these experiments, pupil diameter remained stable and in the range 4–6 mm across subjects. For all five of the subjects tested (PH was unavailable), pupil displacement from the visual axis was less than 0.4 mm. For two subjects (LT, XZ) the measured value was not significantly different from zero (t-test, $P = 0.05$) and for the remaining three subjects, pupil displacement was in the temporal direction. Subject LT also participated in the experiments of Simonet and Campbell (1990), who confirmed under quite different experimental conditions that his pupil decentration was not significantly different from zero.

To estimate the expected magnitude of transverse chromatic aberration for foveal vision, the measured displacement d of the pupil from the visual axis was multiplied by the slope of the appropriate data curve of Fig. 4 and the results displayed in column 7 of Table 1. For four out of five subjects tested, the aberration was less than 1 min and thus about the same as vernier acuity under the conditions of these experiments. The sole exception was MC, for whom we determined a foveal aberration of 1.67 min. However, this exceptional value may be an overestimate since MC was also a subject in the study of Simonet and Campbell (1990), who reported a smaller pupil displacement and smaller transverse chromatic aberration, albeit for different wavelengths. This is probably about the best agreement one could expect of the two experiments, considering that our computed value of the foveal transverse aberration is the product of two empirical estimates (slope and pupil displacement), both of which are subject to experimental error, and that head position in our experiments has an associated uncertainty of about 0.05 mm under the best of conditions.

Longitudinal chromatic aberration

Measurements of the chromatic difference of focus for the two wavelengths of interest (433 nm, 622 nm) were done in conjunction with a more comprehensive study presented elsewhere (Howarth, Zhang, Bradley, Still & Thibos, 1988). The values obtained for the subjects of the current study ranged from 1.2 to 1.8 D as shown in column 4 of Table 1. These estimates tended to be slightly larger than those from the vernier experiment, with an average difference across subjects of 0.29 D for the two experiments. This small discrepancy is probably due to the fact that the best-focus experiment employed the entire dilated pupil whereas the vernier experiment provides a paraxial estimate. As mentioned in Theory, spherical aberration would increase the chromatic difference of focus for rays entering near the margin of the pupil, presumably biasing the results of the best-focus experiment towards higher values This explanation is consistent with previous results of Howarth and Bradley (1986) who used two of the subjects in the present study (AB, PH). They found (see their Fig. 2) that the longitudinal chromatic aberration (interpolated for the wavelength range 433–622 nm) for a dilated pupil was about 0.1–0.2 D greater than the measured aberration obtained with a 3 mm artificial aperture.

DISCUSSION

Comparison of results with theoretical predictions

Many authors have pointed to a simple, reduced-eye model to describe the eye's longitudinal chromatic aberration (Helmholtz, 1909; Ivanoff, 1953; Wald & Griffin, 1947; Bedford & Wyszecki, 1957; Bennett & Rabbetts, 1984; Rog, 1987) and recently the same model has been proposed for representing the transverse chromatic aberration of the eye (Howarth, 1984; Thibos, 1987). In the present paper we have tested this model by using it to predict the experimental results of Figs 4 and 5. There are no free parameters for the theoretical curve shown since its shape is fixed and it is confined to pass through the origin. Similarly, our convention for specifying pinhole displacement ensures that the interpolated data curves will also pass through the origin.

In general, our experimental results closely followed the theoretical predictions of the model. For large displacements of the pinhole, the results suffered from increased variability and significant deviations from the predictions were observed for some subjects. Since the accelerating form of the theoretical curve is traceable to the substantial spherical aberration of the model eye, systematic deviations could be accounted for if the spherical aberration of our subjects' eyes are different from that of the model. If this is the explanation, then different subjects must have had different amounts of spherical aberration because no systematic deviations from the model were evident when data were pooled across subjects (Fig. 5). On the other hand, much of the residual discrepancy between the predicted and measured results could be due to small changes in head posture during the course of the experiment which would have induced errors in pinhole position. Also, slight irregularities in the eye's optical media may have caused localized fluctuations in refractive index and thus more or less chromatic dispersion than expected.

It is perhaps surprising that such a simple optical model accounts so well for the experimental data, given the optical complexity of real eyes with multiple, aspherical refracting surfaces and gradient-index media. One possible explanation is that most of the optical power of the eye, and thus most of its chromatic dispersion, is caused by the initial refracting surface of the cornea. To evaluate such conjectures it should prove useful to compare present results with the predictions of more sophisticated schematic eyes.

Comparison of results with previous experiments

We are aware of only one published, experimental investigation of the magnitude of transverse chromatic aberration across the visual field. Ogboso and Bedell (1987) measured the misalignment of a blue (435 nm peak) and yellow (572 nm peak) vernier target which appeared aligned to the subject when the stimulus was projected with mirrors onto a cylindrical screen extending 60 deg into the peripheral field. The magnitude of the aberration was reported to increase with eccentricity, but not as rapidly as expected from analysis of a wide-angle model eye. Several other features of their experimental results are difficult to reconcile with optical theory. The aberration curve for one of four subjects had even symmetry about the visual axis, rather than the expected odd symmetry. In a control experiment in which both halves of the vernier target were the same color, subjects consistently misaligned the vernier target. This unexplained bias was in many cases quite large and, since they were subtracted from the heterochromatic results, may have caused the aberration to be underestimated. These problematical features of Ogboso and Bedell's results may have been due to the many practical difficulties associated with their experimental method. Targets in the peripheral field will be poorly focussed because of longitudinal chromatic aberration, off-axis aberrations, and peripheral refractive error. Although an attempt was made to correct refractive errors, the introduction of ophthalmic lenses could further confound the experiment by introducing an unknown amount of chromatic aberration. Performance of their subjects may have been hampered by the low, mesopic luminance of the vernier targets (red = 1.16 and blue = 0.06 cd/m^2) and by the uncertainty associated with random interlacing of stimulus eccentricity. When coupled with the poor spatial acuity of the peripheral retina, these many handicaps would make it very difficult for subjects who were not practiced at making vernier judgments in peripheral vision to achieve useful results.

Limitations of the reduced-eye model

One motivation for the present study was to develop an optical model of the eye which adequately describes the magnitude of chromatic aberration and yet is simple enough to provide a useful conceptual framework for analyzing the effects of ocular chromatic aberration on vision. One general area of application would be in predicting the retinal location of images for different wavelengths. As our experiments have shown, large differences of image position which arise because of chromatic dispersion are well described by the reduced-eye model. Another general area of interest is predicting image quality. In a theoretical analysis of paraxial image quality in humans, Van Meeteren (1974) showed that for small pupils the eye is diffraction limited but for large pupils the dominant aberration limiting image quality is chromatic difference of focus. Further analysis has shown that when the entrance aperture is displaced from the visual axis, severe losses of image quality will result because of the effects of transverse chromatic aberration in the eye (Van Meeteren & Dunnewold, 1983; Thibos, 1987). Consequently, if chromatic aberration is the primary determinant of image quality for both centered and eccentric pupils, and since the reduced eye provides a good account of ocular chromatic aberration, then it is reasonable to expect that this simple model should prove useful for predicting retinal image quality.

Present experimental support for the application of the reduced-eye model to problems of peripheral vision remains indirect. Here we measured the magnitude of transverse chromatic aberration for foveal imaging through a displaced pinhole, whereas peripheral vision is normally through a nearly centered, natural pupil. Although the two cases are formally equivalent for the model eye (see Theory), the actual path of light rays is quite different for real eyes because a centered pupil will confine rays to the middle of the crystalline lens whereas a displaced pinhole will isolate rays passing through marginal portions of the lens. Because of the gradient of refractive index of the human lens (Pierscionek, Chan, Ennis, Smith & Augusteyn, 1988), marginal and central rays may suffer from different amounts of chromatic dispersion. On the other hand, this may be a minor effect in human eyes since the majority of optical power is contributed by the cornea (Campbell, 1989). Another uncertainty in applying the model to peripheral vision is that equation (3) contains one free parameter, NP, the distance between the entrance pupil and nodal point (see Fig. 2). The assumed value of 3.98 mm has not yet been subjected to experimental test. Nor have we taken into account the possibility that this distance may be wavelength dependent or subject to individual variability.

Foveal transverse chromatic aberration and angle ψ

We argue that transverse chromatic aberration will occur for foveal viewing if the pupil is not centered on the visual axis. Such decentering causes the visual and achromatic axes, as defined for the model eye in Theory, to be inclined at an angle ψ which, from the geometry of Fig. 6, is given by:

$$\sin(\psi) = d/NP. \qquad (7)$$

This equation is a special case of the more general concept of equivalent eccentricity, introduced in Theory, which provides the following alternative way of thinking about the effect of a decentered pupil. Displacement of the pupil from the visual axis induces transverse chromatic aberration for foveal viewing which is equal in magnitude to that present for an object located at some eccentricity ψ from the visual axis. Thus, by using equation (7) we may infer the angle ψ from our measurements of pupil displacement and the results are given in column 6 of Table 1. For two subjects (LT, XZ) estimates of ψ were not significantly different from zero and the average for all subjects was 2 deg, with the pupil displaced from the visual axis in the temporal direction.

Our estimates of the magnitude of transverse chromatic aberration for foveal vision agree with experimental measurements by others (Hartridge, 1947; Simonet & Campbell, 1988, 1990) but are considerably less than previously calculated on theoretical grounds (Le Grand, 1967; Howarth, 1984; Thibos, 1987). We suspect that prior theoretical predictions were overestimates because they were based on the angle α (about 5 deg; Tscherning, 1920) between the visual axis and the optical axis of the eye, traditionally identified as the line containing the centers of curvature of the eye's refracting surfaces. We argue here that the angle ψ between visual and achromatic axes is more useful for predicting the effects of optical aberrations of the eye, such as transverse chromatic aberration, because it takes account of the key role played by the pupil in determining the angle of incidence of those rays which will pass through to stimulate the retina. The angle of incidence, in turn, determines the angle of refraction and chromatic dispersion according to Snell's law

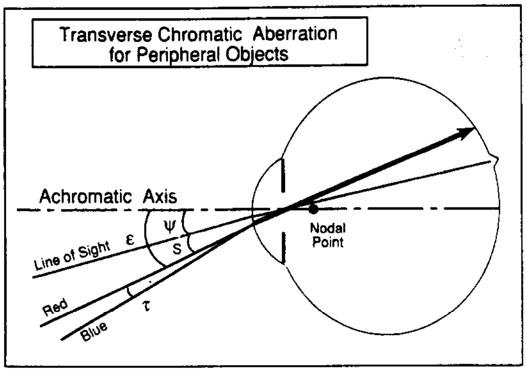

Fig. 7. Transverse chromatic aberration for peripheral objects depends directly on ε, the eccentricity of the stimulus from the achromatic axis, which is seen to be the sum of two components: the eccentricity s of the stimulus from the foveal line-of-sight and the angle ψ between the line-of-sight and the achromatic axis.

and hence the magnitude of ocular transverse chromatic aberration. This change of reference axis would bring theory and measurement into closer agreement since our calculated values of ψ are substantially less than the commonly accepted values of α. We cannot be certain that this is the full explanation, however, because of our uncertainty regarding the assumed value for factor NP (the distance between pupil and nodal point) used to compute ψ by equation (7). Nevertheless, our finding that angle ψ is essentially zero for two subjects indicates that, at least for some individuals, the pupil is closely centered on the visual axis, thereby reducing transverse chromatic aberration to nearly zero for foveal vision.

Predicted transverse chromatic aberration for peripheral vision

Functionally, the fovea lies at the center of the visual field but optically, the center of symmetry for transverse chromatic aberration is set by the achromatic axis. To unify these visual and optical reference frames is, in general, a three dimensional problem since the stimulus may lie anywhere in visual space. Here we use the reduced-eye model to analyze the simpler two-dimensional case where the stimulus is confined to lie in the plane formed by the visual and achromatic axes, which is the plane of the diagram of Fig. 7. In practice, stimulus eccentricity s is referenced to the fovea by defining eccentricity as the angle between the chief ray of the stimulus and the line-of-sight. Since the foveal line-of-sight nearly coincides with the visual axis for distant objects, negligible error is introduced by asserting that it too intersects the achromatic axis with angle ψ. Therefore, the total eccentricity ε of the stimulus relative to the achromatic axis is given by the algebraic sum $\varepsilon = s + \psi$, taking care to maintain an appropriate sign convention. Given ε, it is then a simple matter to calculate the expected magnitude of transverse chromatic aberration for stimuli in the peripheral field. First, one applies equation (1) to calculate θ, the angle of incidence of the chief ray. Snell's law then gives the angles of refraction for the two wavelengths in question and the difference of these angles is, by definition, the magnitude of transverse chromatic aberration of the model eye and thus, by inference, the magnitude of the eye's aberration for the given location in the peripheral field.

Implications for vision

The optical system of the eye is an image-degrading filter located at the input of the visual system and thus represents a potential limiting factor for any visual task. If chromatic aberration is one of the primary determinants of

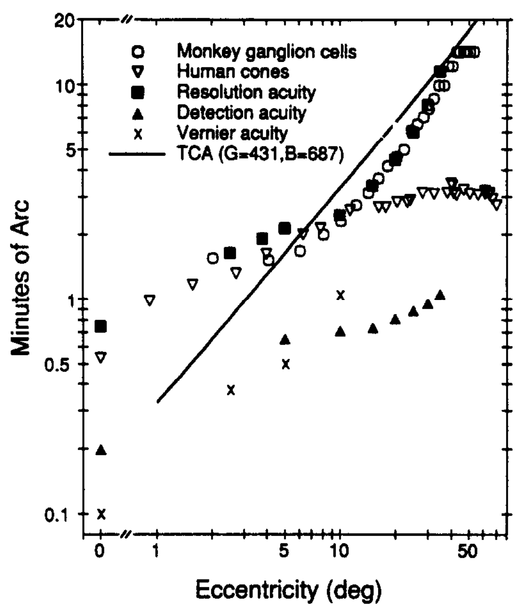

Fig. 8. Comparison of the magnitude of transverse chromatic aberration with dimensions of the retinal mosaic and with psychophysical thresholds of spatial vision. All dimensions are referenced to object space. Theoretical curve shows the magnitude of the aberration for the reduced-eye model ($\psi = 0$) between the extremes of the visible spectrum marked by the Fraunhofer G-line (434 nm) and B-line (687). Open symbols show the characteristic spacing of retinal neurons determined anatomically. Solid symbols show the minimum angle of resolution and the minimum angle of detection for luminance gratings produced interferometrically, and also the minimum angle of offset for detecting misalignment of two vertical dots. See text for sources of data.

optical quality, then we should expect to see its signature in the optical limits to human visual performance. Evidence to this effect is reviewed briefly further on.

To help gauge the potential significance of transverse chromatic aberration for visual function, consider the tiny spectrum produced on the retina by a point source of white light located in the peripheral field. The question arises, is the width of this spectrum comparable in magnitude with the spatial dimensions of the retinal mosiac, or of other visual functions? To approach this question we have replotted in Fig. 8 the smallest threshold values reported in the literature for trained subjects tested for three types of spatial acuity: resolution acuity (minimum angle of resolution for luminance gratings, Thibos, Cheney & Walsh, 1987b; Williams & Coletta, 1987) detection acuity (minimum angle of detection for luminance gratings, Williams, 1985; Thibos et al., 1987b) and vernier acuity (two-dot vernier, Westheimer, 1982). Also included are anatomical measurement of cell spacing for two classes of retinal neurons: cone photoreceptors (Osterberg, 1935) and beta ganglion cells (Perry, Oehler & Cowey, 1984). To calculate transverse chromatic aberration of the reduced eye with pupil centered ($\psi = 0$) on the visual axis, the angular dispersion of the visible spectrum was calculated between the Fraunhofer G (434 nm) and B (687 nm) lines, i.e. the wavelengths for which the spectral sensitivity of the CIE standard observer falls to 1%. All values shown in Fig. 8 are referenced to object space.

When compared to the dimensions of the retinal mosaic, we see that near the fovea the aberration is less than the distance between neighboring cones. Beyond about 5 deg of eccentricity the entire spectrum would just span the distance between neighboring ganglion cells, which implies that the individual colored bands could not be resolved by this array of neurons. However, the same spectrum would cover several cones. The psychophysical comparisons drawn in Fig. 8 show that within the central 5 deg of the visual field the optical aberration is less than the minimum angle of resolution but is greater than vernier acuity. Beyond 5 deg the angular subtense of the colored fringes is nearly the same as resolution threshold but is well above the threshold for detection of spatial contrast and for vernier alignment. Although the available evidence is insufficient for judging whether the fringes ought to be visible in peripheral vision, it may be that for many observers the spread of the color fringes is below the perceptual threshold, which would explain why the aberration has not been widely studied previously.

Foveal vision. Campbell (1958) found that when a grating test object was viewed through a 1 mm pinhole decentered by 4 mm, an eightfold loss of visual acuity occurred, which he attributed to neural factors. Subsequent experiments by Campbell and Gregory (1960) indicated that some of that acuity loss should be attributed instead to optical factors. Green (1967) repeated those experiments and found a threefold loss of acuity for polychromatic targets viewed through a displaced pinhole aperture which he attributed wholly to optical factors. Further theoretical analysis by Van Meeteren and Dunnewold (1983) led them to support Green's conclusion while further refining the argument by showing that the major optical aberration affecting image contrast for a small, displaced pupil is transverse chromatic aberration. Further support for this chromatic aberration hypothesis was provided by the calculations of Thibos (1987) based on the same reduced-eye model presented here. He showed that transverse chromatic aberration alone is sufficient to account for Green's results. Additional analysis of the model eye has correctly predicted that a similar loss of acuity should occur even when the polychromatic retinal stimulus is produced interferometrically, a technique which avoids the contrast losses expected from longitudinal chromatic aberration and other defects of focus (Thibos et al., 1987a). Acuity losses also occur when measured with Maxwellian-view clinical instruments that are employed off-axis (Thibos, Bradley & Still, 1989). Once again, the visual loss is predicted by ocular chromatic aberration. Thus there is mounting evidence under a variety of circumstances that the transverse chromatic aberration of the eye is potentially a major limiting factor for foveal vision through a decentered aperture. This conclusion re-emphasizes the importance of accurate centering of the natural pupil on the visual axis to maximize image quality.

Peripheral vision. In peripheral vision, resolution acuity is limited by the neural architecture of the retina (Thibos et al., 1987b) rather than by optical image quality (Smith & Cass, 1987; Still & Thibos, 1987). However, patterns too fine to be resolved may still be detected up to very high spatial frequencies but they are misperceived as "aliases" of the stimulus (Thibos, Walsh & Cheney, 1987c). This raises the possibility that detection acuity (i.e. the finest visible pattern) may be limited by the contrast-attenuating effects of transverse chromatic aberration. Based on an analysis of the reduced-eye model, Cheney and Thibos (1987) reasoned that the loss of image contrast will vary with grating orientation and will be zero when the grating is parallel to the visual meridian of the stimulus. Their experimental results confirmed the presence of orientational anisotropy for detection of peripheral gratings, thus supporting the idea that transverse chromatic aberration can be an important factor which affects contrast detection in the periphery.

Binocular vision. Image displacement caused by transverse chromatic aberration has been implicated in chromostereopsis, a perceptual illusion in which differently colored objects appear to be at different depths even though they lie in the same plane (for reviews, see Bennett & Rabbetts, 1984; Allen & Rubin, 1981). Einthoven (1885) proposed that if the pupils of the two eyes are displaced temporally from the visual axis, then blue rays from a point source will intersect the retinae on the nasal side of red rays from the same source. This induced binocular disparity will make the blue rays appear to come from a source more distant than the source of the red rays. Given the model eye proposed in this paper, we may still compute the expected magnitude of the induced binocular disparity by considering the chromatic aberration of the eye in image space, as in Fig. 1A. Assuming the two eyes are identical, the visual angle subtended by the disparate red and blue images, when referenced to the nodal point, is greater than the angle between chief rays by the ratio of the focal distance to the posterior nodal distance, which for the reduced-eye is 4/3. The total disparity for the two eyes will be double this monocular value, or 8/3 times the transverse chromatic aberration of each eye alone. Judging from our results, this could amount to several minutes of arc of binocular disparity at the fovea, which is well above threshold for the subjective perception of depth in central vision.

A curious, unresolved feature of chromostereopsis is the reversal of apparent depth of the red and blue targets that occurs when illumination levels are reduced, which is consistent with Ogboso and Bedell's (1987) finding of negative transverse chromatic aberration for foveal vision at low levels of illumination. To explain the stereoscopic reversal, Kishto (1965) suggested the center of the entrance pupil shifts nasally as the iris dilates, eventually causing a change of sign for angle ψ. Others (Vos, 1966; Sundet, 1976) have suggested an alternative explanation based on the Stiles–Crawford effect. By this explanation, dilation of the pupil admits marginal rays which have differential visibility according to the angle with which they strike the retinal surface. If the rays of maximum visibility are nasally eccentric in the pupils, then the result would be a shift of the effective center of the blur circle away from the geometrical center in the temporal direction. If this shift is large enough, the illusion would reverse in sign even though the achromatic and visual axes remain fixed. This is an intriguing hypothesis since it challenges a fundamental tenet of visual optics, that the perceived visual direction of a blurred image is determined by the chief ray.

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APPENDIX

The functional form of the smooth curves in Figs 4, 5 and 8 is given by the following algorithm. See Fig. 3 for graphical definition of variables.

Purpose: Procedure CHROMO computes chromatic aberration in object space of reduced eye for foveal vision through displaced pupil.

Usage: CALL CHROMO (H, SW,LW, TRANS, LONG).

Method: Ivanoff (1953) construction with constraint that exit rays are coincident and strike the image plane at the visual axis. Index of refraction for water is by Cornu's formula (Le Grand, 1967).

Arguments:

H = distance QM from visual axis to point of intersection of chief rays at refracting surface, in mm.
LW = long wavelength, in micrometers.
SW = short wavelength, in micrometers.
$TRANS$ = returned value of transverse chromatic aberration, in min. arc.
$LONG$ = returned value of longitudinal chromatic aberration, in diopters.

Parameters:

n_lw = index of refraction for long wavelength.
n_sw = index of refraction for short wavelength.
a = angle between visual axis and incident chief ray for long wavelength.
b = angle between visual axis and incident chief ray for short wavelength.
γ = angle between visual and refracted chief rays.
ϕ_lw = angle of incidence for chief ray of long wavelength.
ϕ_sw = angle of incidence for chief ray of short wavelength.
β = angle of refraction for both exit chief rays.
MN = axial distance from point of intersection to nodal point.
NF = distance from nodal point to posterior focal plain.
R = radius of curvature of refracting surface.
v_lw = vergence of conjugate point for long wavelength.
v_sw = vergence of conjugate point for short wavelength.

```
Procedure (H, SW, LW, TRANS, LONG):
/***** SINGULAR CASES AND INITIALIZATION *****/
if (H = 0) then begin;
  TRANS = 0.0;
  return;                                /* LONG is indeterminant */
  end;
NF = 16.67;                              /* reduced-eye constant */
R = 5.55;                                /* reduced-eye constant */
min_per_rad = 60*57.29578;               /* for converting radians to minutes of arc */
n_lw = 1.31848 + 0.0066620/(lw - 0.1292); /* Cornu's formula */
n_sw = 1.31848 + 0.0066620/(sw - 0.1292); /* Cornu's formula */

/**** COMPUTE TRANSVERSE CHROMATIC ABERRATION ****/
MN = sqrt(R*R - H*H);
γ = atan[H/(NF + MN)];
sin_β = NF*sin(γ)/R;                     /* law of sines triangle NQF */
φ_sw = asin(n_sw*sin_β);                 /* Snell's law for short wavelength */
φ_lw = asin(n_lw*sin_β);                 /* Snell's law for long wavelength */
TRANS = min_per_rad*(φ_sw - φ_lw);       /* transverse chromatic aberration = angle between chief rays */

/**** COMPUTE LONGITUDINAL CHROMATIC ABERRATION ***/
β = asin(sin_β);
b = φ_sw - β - γ;
v_sw = 1000*tan(b)/H;                    /* vergence for short wavelength */
a = φ_lw - β - γ;
v_lw = 1000*tan(a)/H;                    /* vergence for long wavelength */
LONG = v_sw - v_lw;                      /* longitudinal aberration = vergence difference */
end;
```